Liquid-Crystal State of $\nu = 1/m$ Quantum Hall Effects

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At filling factor $\nu = 1/m$, m odd integer, I present variational ground-state and excited-state wave functions, of two-dimensional electron system with homogeneous ion background, that show the condensation into a liquid-crystal state. For $m = 1, 3, 5$, the ground-state energy per electron is substantially lower than the Laughlin one, for uniform liquid state.

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Discovery of the fractional quantum Hall effect [1] in two-dimensional electron systems (2DES) of GaAs-based samples and the Laughlin seminal theory of this effect [2] have generated strong interest to properties of fractional quantum Hall states at $\nu = 1/m$, especially for $m = 3$ and 5, [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. Current understanding is that for $m = 3,\ 5$ the Laughlin wave function [2] gives the best known analytical approximation of exact many-body ground-state wave function [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. For $m = 1$ the Laughlin wave function coincides with the Hartree-Fock approximation (HFA) one [2, 17], built from the symmetric gauge single-electron wave functions of the lowest Landau level, and leads to the total energy per electron [2] $\epsilon_{HF} = -\sqrt{\pi/8}e^2/(\varepsilon\ell_0)$; $\ell_0 = \sqrt{\hbar c/|e|B}$ is the magnetic length and ε the background dielectric constant. In present study strong many-body effects are essentially related as with N electrons of 2DES so with N ions. I treat the ions on more equal footing with 2DES than previously [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19]. In addition, I use more adequate sets of single-body wave functions; they are localized mainly (or exactly) within the unit cell $L_x^{\square} \times L_x^{\square}$, $(L_x^{\square})^2 = L_x L_y/N$. These wave functions help better reflect the tendency: i) of an ion to be mainly localized within its own unit cell, and ii) of an electron to be present mainly within any such unit cell, with equal probability.

In this Letter, at filling factors $\nu = 1/m$ with odd integer m, I present many-body variational ground-state and excited-state wave functions for electron-ion system, with homogeneous ion density, that have strong correlations between 2DES and ions. The former wave function result in: i) substantially lower ground-state energies for $\nu = 1,\ 1/3,\ 1/5$ than obtained in Ref. [2]; ii) the electron density, Eq. (10), periodic along one direction with period $\sqrt{2\pi/m}\ \ell_0$ is typically very weakly modulated; iii) fractionally quantized Hall conductance, for $m = 3,\ 5, \ldots$. I obtain finite excitation gaps along with fractional quasielectron, e/m, and quasihole, $|e|/m$, charges, for $m \geq 3$.

We consider a zero-thickness 2DES of width L_y ($L_y/2 > y > -L_y/2$) and of length L_x ($L_x > x > 0$) in the presence of a magnetic field, $\mathbf{B} = B\hat{\mathbf{z}}$. The Landau gauge for the vector potential, $\mathbf{A}(\mathbf{r}) = (-By, 0, 0)$, is used; N electrons of a 2DES and N ions are located in the main region, $L_x \times L_y$. As ions are very heavy, their kinetic energy can be neglected [2, 17]. Then the many-body Hamiltonian $\hat{H} = \hat{H}_0 + V_{ee} + V_{ei} + V_{ii}$, where the kinetic energy of electrons $\hat{H}_0 = \sum_{i=1}^{N} \hat{h}_{0i}$, $\hat{h}_{0j} = [i\hbar\boldsymbol{\nabla}_j + e\mathbf{A}(\mathbf{r}_j)/c]^2/2m^*$; the electron-electron potential $V_{ee} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} V(\mathbf{r}_i - \mathbf{r}_j)$, $\mathbf{r}_i = (x_i, y_i)$; the electron-ion potential $V_{ei} = -\sum_{i=1}^{N}\sum_{j=1}^{N} V(\mathbf{r}_i - \mathbf{R}_j)$; and the ion-ion potential $V_{ii} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} V(\mathbf{R}_i - \mathbf{R}_j)$, $\mathbf{R}_i = (X_i, Y_i)$; $V(\mathbf{r}) = e^2/\varepsilon|\mathbf{r}|$.

Let us for a $x-$stripe region, given by $x \in (L_x^{\square}(n_{xs}^{\alpha} - 1); L_x^{\square} n_{xs}^{\alpha})$ and $y \in (-\infty, \infty)$, introduce normalized solutions of the single-electron Schrödinger ($\omega_c = |e|B/m^*c$, $k_{x\alpha} = 2\pi n_{ys}^{\alpha}/L_x^{\square}$) equation

$$\hat{h}_0 \psi^{L_x^{\square}}_{n_\alpha; n_{xs}^{\alpha}, k_{x\alpha}}(\mathbf{r}) = \hbar\omega_c(n_\alpha + 1/2)\psi^{L_x^{\square}}_{n_\alpha; n_{xs}^{\alpha}, k_{x\alpha}}(\mathbf{r}) \quad (1)$$

of ($\Psi_n(y)$ is the harmonic oscillator function) the form

$$\psi^{L_x^{\square}}_{n_\alpha; n_{xs}^{\alpha}, k_{x\alpha}}(\mathbf{r}) = \frac{e^{ik_{x\alpha}x}}{(L_x^{\square})^{1/2}} \Psi_{n_\alpha}(y - y_0(k_{x\alpha})), \quad (2)$$

where $y_0(k_{x\alpha}) = \ell_0^2 k_{x\alpha}$, $n_{ys}^{\alpha} = 0, \pm 1, \ldots, \pm(n_{ys}^{\max,t} - 1)/2$; $n_{ys}^{\max,t}$ is the odd integer such that $(2\pi/L_x^{\square}) n_{ys}^{\max,t} \ell_0^2 = L_y$. For $x > L_x^{\square} n_{xs}^{\alpha}$ or $x < L_x^{\square}(n_{xs}^{\alpha} - 1)$, $\psi^{L_x^{\square}}_{n_\alpha; n_{xs}^{\alpha}, k_{x\alpha}}(\mathbf{r}) \equiv 0$. Here $n_{xs}^{\alpha} = 1, 2, \ldots, n_{xs}^{\max}$ gives the number to the $x-$stripe region and $L_x^{\square} n_{xs}^{\max} = L_x$. Then the total number of states of the wave functions Eq. (2), on the $n_\alpha-$th Landau level in the main region, is $n_{xs}^{\max} n_{ys}^{\max,t} = L_x L_y/(2\pi\ell_0^2) = N_L$, which is equal to the number of states of "usual" wave functions [20] $\psi^{L_x}_{n_\alpha; 1, k_{x\alpha}}(\mathbf{r})$. Wave functions Eq. (2) are orthonormal as

$$\int_0^{L_x} dx \int_{-\infty}^{\infty} dy\ \psi^{L_x^{\square}*}_{n_\beta; n_{xs}^{\beta}, k_{x\beta}}(\mathbf{r}) \psi^{L_x^{\square}}_{n_\alpha; n_{xs}^{\alpha}, k_{x\alpha}}(\mathbf{r}) = \delta_{n_\beta, n_\alpha} \delta_{n_{xs}^{\beta}, n_{xs}^{\alpha}} \delta_{k_{x\beta}, k_{x\alpha}}. \quad (3)$$

It can be shown that the set of single-electron wave functions Eq. (2) is complete. Eq. (3) reduces to well known

result [20] for $\psi^{L_x}_{n_\alpha;1,k_{x\alpha}}(\mathbf{r})$ if to change $L_x^\square$ on L_x. Further, $(L_x^\square)^2 = L_xL_y/N = L_xL_y/\nu N_L$, where N is fixed for a given sample; hence, $L_x^\square$ is also fixed. Then we obtain

$$L_x^\square/\Delta y_0 = 1/\nu. \quad (4)$$

From (4) it is seen that within each unit cell can appear only an odd integer number, $m = 1, 3, \ldots$, of quantized oscillator centres, $y_0(k_{x\alpha})$; even m, not treated here, is a special case. Then Eq. (4) gives, $\ell = 0,\ 1, \ldots$, that

$$1/\nu = m, \quad (5)$$

where $m = 2\ell + 1$. From Eqs. (4), (5) it follows

$$L_x^\square = \sqrt{2\pi m}\ \ell_0. \quad (6)$$

As for $\nu = 1/m$ there are m quantized values of $y_0(k_{x\alpha i})$ within an $i-$th unit cell and each of them has a particular position within the unit cell, we separate all N_L states Eq. (2), of a $n_\alpha-$th Landau level, into the m sets of wave functions. Within any such $n-$th set of states $[y_0(k^{(n)}_{x\alpha j}) - y_0(k^{(n)}_{x\alpha i})] = kL_x^\square$, where k is an integer. Here $j(i)$ is the number of a unit cell; it can be any integer from 1 to N. This $i-$number exactly defines the $i-$th unit cell. The superscript in $k^{(n)}_{x\alpha i}$ is given to distinguish the $k_{x\alpha i}$ pertinent to the $n-$th set of states; the subscript (superscript), i, in $k_{x\alpha i}$, etc. ($n^{(i)}_{xs}$, etc.) indicates belonging to the $i-$th unit cell. We choose the values of n as $n = 0, \ldots, \pm\ell$ and define $k^{(n)}_{xi} \equiv k^{(n)}_{x\alpha i}$ as

$$k^{(0)}_{xi} = (2\pi m/L_x^\square)n^{(i)}_{ys}, \ldots, k^{(\pm\ell)}_{xi} = k^{(0)}_{xi} \pm 2\pi\ell/L_x^\square, \quad (7)$$

where $n^{(i)}_{ys} = 0, \pm1, \ldots, \pm(n^{\max}_{ys} - 1)/2$; $n^{\max}_{ys} = n^{\max,t}_{ys}/m$ is an odd integer. Wave functions Eq. (2) of the $n_\alpha = 0$ Landau level we denote, at $\nu = 1/m$, as well as

$$\varphi^{i,(m)}_{n^{(i)}_{xs},k^{(n)}_{xi}}(\mathbf{r}) \equiv \psi^{L_x^\square}_{0;n^{(i)}_{xs},k^{(n)}_{xi}}(\mathbf{r}). \quad (8)$$

I assume the ground-state wave function of electron-ion system, $\Psi^{(m)}_{N,N}(\mathbf{r}_1, \ldots, \mathbf{r}_N; \mathbf{R}_1, \ldots, \mathbf{R}_N)$, in the form ($C_n \equiv C_n(m)$)

$$\Psi^{(m)}_{N,N} = \left[\sum_{n=-\ell}^{\ell} C_n \Psi^{n,(m)}_N\right] \prod_{i=1}^{N} \phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R}_i), \quad (9)$$

where $|C_n|^2 = 1/m$, and the "partial" many-electron wave function, $\Psi^{n,(m)}_N(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$, is $N-$dimensional Slater determinant of wave functions Eq. (8). Here "single-ion" wave functions $\phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R})$ are introduced as: $|\phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R})|^2 = 1/(L_x^\square)^2$, if both $X \in (L_x^\square(n^{(i)}_{xs} - 1), L_x^\square n^{(i)}_{xs})$ and $Y \in (L_x^\square(n^{(i)}_{ys} - 1/2), L_x^\square(n^{(i)}_{ys} + 1/2))$; if X or/and Y is outside of this $i-$th unit cell then $\phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R}) \equiv 0$. The set of these single-body wave functions is orthonormal; then, $\langle\Psi^{(m)}_{N,N}|\Psi^{(m)}_{N,N}\rangle = 1$. The electron density, $n(\mathbf{r}) = \langle\Psi^{(m)}_{N,N}|\sum_{j=1}^{N}\delta(\mathbf{r}-\mathbf{r}_j)|\Psi^{(m)}_{N,N}\rangle$, in the main region, for $n^{\max}_{ys} \gg 1$ (then $N \gg 1$), is

$$n(y) = \frac{\ell_0^{-2}}{2\pi m}[1 + 2\sum_{k=1}^{\infty} e^{-\pi m k^2/2}\cos(\frac{\sqrt{2\pi m}}{\ell_0}ky)], \quad (10)$$

after using the Fourier transformations and the Poisson's summation formula [21]. Eq. (10) gives that a unit cell is "dressed" by electron charge, e. The ion density $n_{io}(\mathbf{r}) = n_{io}$, where $n_{io} = 1/(2\pi m\ell_0^2)$. Point out, in a good approximation of experimental conditions, $\Psi^{(m)}_{N,N}$ gives that each ion is located in its own unit cell.

The total energy in the ground-state Eq. (9) is

$$E^{(m)}_N = \langle\Psi^{(m)}_{N,N}|\hat{H}|\Psi^{(m)}_{N,N}\rangle, \quad (11)$$

where the kinetic energy term gives $m^{-1}\sum_{n=-\ell}^{\ell}\langle\Psi^{n,(m)}_N|\hat{H}_0|\Psi^{n,(m)}_N\rangle = \hbar\omega_c N/2$, cf. with Ref. [8]; details will be published elsewhere [22]. In Eq. (11) the term $\langle\Psi^{(m)}_{N,N}|V_{ii}|\Psi^{(m)}_{N,N}\rangle$ obtains the form

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{-\infty}^{\infty}d\mathbf{R}\int_{-\infty}^{\infty}d\mathbf{R}'\frac{e^2}{\varepsilon|\mathbf{R}-\mathbf{R}'|} \times|\phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R})|^2\ \ |\phi^{(j)}_{n^{(j)}_{xs},n^{(j)}_{ys}}(\mathbf{R}')|^2. \quad (12)$$

$\langle\Psi^{(m)}_{N,N}|V_{ei}|\Psi^{(m)}_{N,N}\rangle$ in Eq. (11) has the form

$$-\frac{1}{m}\sum_{n=-\ell}^{\ell}\sum_{i=1}^{N}\sum_{j=1}^{N}\int_{-\infty}^{\infty}d\mathbf{r}\int_{-\infty}^{\infty}d\mathbf{R}\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{R}|} \times|\varphi^{i,(m)}_{n^{(i)}_{xs},k^{(n)}_{xi}}(\mathbf{r})|^2\ |\phi^{(j)}_{n^{(j)}_{xs},n^{(j)}_{ys}}(\mathbf{R})|^2. \quad (13)$$

$\langle\Psi^{(m),eh}_{N,N}|V_{ee}|\Psi^{(m),eh}_{N,N}\rangle$ in Eq. (11) we can rewrite as

$$\frac{1}{2m}\sum_{n=-\ell}^{\ell}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\langle\Psi^{n,(m)}_N|\frac{e^2}{\varepsilon|\mathbf{r}_i-\mathbf{r}_j|}|\Psi^{n,(m)}_N\rangle, \quad (14)$$

where the matrix elements are calculated as in HFA [23]. Then Eq. (11) is written, $\tilde{E}^{(m)}_N = E^{(m)}_N - \hbar\omega_c N/2$, as

$$\tilde{E}^{(m)}_N = \frac{e^2N}{\varepsilon\ell_0}\left[F_2^A(m) + F_1^C(m) + \Delta F_1^C(m)\right], \quad (15)$$

where $F_2^A(m)$ is exchange-alike term from Eq. (14),

$$F_2^A(m) = -\frac{1}{\pi}\sum_{k=-\infty;k\neq0}^{\infty}e^{-\pi m\,k^2}\int_0^\infty d\xi\int_0^\infty d\eta \times e^{-\eta^2/2}\ S_m^2(\xi)\ G_m(\xi,\eta;k), \quad (16)$$

$G_m(\xi,\eta;k)=[(\xi-\sqrt{2\pi m}\ k)^2+\eta^2]^{-1/2}$. Further, from the "diagonal" part, $i=j$, of Eq. (13) it follows the term

$$F_1^C(m) = -\frac{2}{\pi}\int_0^\infty d\xi\int_0^\infty d\eta e^{-\eta^2/4}\ (\xi^2+\eta^2)^{-1/2} \times f_m(\eta)\ S_m(\eta)\ S_m^2(\xi), \quad (17)$$

where $S_m(x)=\sin(\sqrt{\pi m/2}\ \ x)/(\sqrt{\pi m/2}\ \ x)$, $f_1(\eta)=1$; for $m=3,5,\ldots$, $f_m(\eta)=m^{-1}\left[1+2\sum_{n=1}^{\ell}\cos(\sqrt{2\pi/m}\ n\ \eta)\right]$. The sum of i) "nondiagonal" part, $i\neq j$, of Eq. (13), ii) Eq. (12), and iii) direct-alike contribution from Eq. (14) gives

$$\Delta F_1^C(m) = -\frac{1}{\pi}\int_0^\infty d\xi\int_0^\infty d\eta\ g_m(\eta)S_m^2(\xi)/\sqrt{\xi^2+\eta^2} +(1/\sqrt{2\pi\ m})\sum_{k=1}^{\infty}k^{-1}\ e^{-\pi\ k^2/m}, \quad (18)$$

where $g_m(\eta)=S_m^2(\eta)+e^{-\eta^2/2}-2e^{-\eta^2/4}\ f_m(\eta)\ S_m(\eta)$. We can rewrite Eq. (15) as $\tilde{E}_N^{(m)}/[e^2N/(\varepsilon\ell_0)]=U^C(m)$, where $U^C(m)=[F_1^C(m)+\Delta F_1^C(m)+F_2^A(m)]$ gives lowering of the total energy per electron in the units of $e^2/\varepsilon\ell_0$. I calculate numerically that $U^C(1)\approx-1.202775$, $U^C(3)\approx-0.712971$, $U^C(5)\approx-0.552704$, and $U^C(7)\approx-0.466528$. Here $U^C(1)$, $U^C(3)$, and $U^C(5)$ are substantially lower than pertinent total lowering at $\nu=1$, $1/3$, and $1/5$ for the Laughlin variational wave function [2] $-\sqrt{\pi/8}\approx-0.6267$, -0.4156 ± 0.0012, and -0.3340 ± 0.0028, respectively. Notice, for $m=1$, if in Eqs. (11)-(14) formally to change both single-electron and "single-ion" wave functions on "usual" single-particle wave function $\psi^{L_x}_{0;1,k_{x\alpha}}$, then $\tilde{E}_N^{(m)}=N\epsilon_{HF}$, for $L_x\to\infty$.

I assume, $\Psi^{i_0;j_0}_{N,N;(m)}(\mathbf{r}_1,\ldots,\mathbf{r}_N;\mathbf{R}_1,\ldots,\mathbf{R}_N)$, excited-state wave function of the ground-state Eq. (9) as

$$\Psi^{i_0;j_0}_{N,N;(m)}=\prod_{i=1}^{N}\phi^{(i)}_{n^{(i)}_{xs},n^{(i)}_{ys}}(\mathbf{R}_i)\sum_{n=-\ell}^{\ell}\tilde{C}_n[(1-\delta_{n,0}) \times\Psi_N^{n,(m)}(\mathbf{r}_1,\ldots)+\delta_{n,0}\Phi^{i_0;j_0}_{N;(m)}(\mathbf{r}_1,\ldots)], \quad (19)$$

where $\tilde{C}_n=C_n$, for $n\geq0$, and $\tilde{C}_n=-C_n$, for $n<0$. An excited "partial" many-electron wave function $\Phi^{i_0;j_0}_{N;(m)}$ it follows from the $\Psi_N^{0,(m)}$ after changing of the i_0-th row, $\varphi^{i_0,(m)}_{n^{(i_0)}_{xs},k^{(0)}_{xi_0}}(\mathbf{r}_1),\ldots,\varphi^{i_0,(m)}_{n^{(i_0)}_{xs},k^{(0)}_{xi_0}}(\mathbf{r}_N)$, by the determinant row of the, for $m\geq3$, form $\varphi^{j_0,(m)}_{n^{(j_0)}_{xs},k^{(\tilde{n})}_{xj_0}}(\mathbf{r}_1),\ldots,\varphi^{j_0,(m)}_{n^{(j_0)}_{xs},k^{(\tilde{n})}_{xj_0}}(\mathbf{r}_N)$, where $\tilde{n}\neq0$; Eq. (7) gives $k^{(\tilde{n})}_{xj_0}=(2\pi\ m/L_x^{\square})\left[n^{(j_0)}_{ys}+\tilde{n}/m\right]$, $\tilde{n}=\pm1,\ldots,\pm\ell$; i.e., $k^{(\tilde{n})}_{xj_0}\neq k^{(0)}_{xi}$, where $i=1,\ldots,N$. For $m=1$ in new determinant row: i) the implicit spin up wave function $|1>=\psi_1(\sigma_j)$ should be substituted by spin down one, $|-1>=\psi_{-1}(\sigma_j)$; ii) $\tilde{n}=0$. We assume that the i_0-th unit cell, where the quasihole appears, as well as the j_0-th unit cell, where the quasielectron is mainly localized, there are well inside of the main region. $\Psi^{i_0;j_0}_{N,N;(m)}$ describes excitation of a quasiexciton type [2, 3, 4, 5, 11, 12]. It is seen that at any separation between the quasielectron and the quasihole their charges are given (details will be published elsewhere [22]) as e/m and $|e|/m$, respectively. We need the energy gap, $\Delta^{(m)}$, for the creation of one quasielectron and one quasihole, infinitely spatially separated[2, 3, 4, 5, 11, 12, 14, 16]; notice, $\langle\Psi^{i_0;j_0}_{N,N;(m)}|\Psi^{(m)}_{N,N}\rangle=0$ and $\langle\Psi^{i_0;j_0}_{N,N;(m)}|\Psi^{i_0;j_0}_{N,N;(m)}\rangle=1$.

With infinitely spatially separated quasielectron and quasihole, $\Delta^{(m)}=\langle\Psi^{i_0;j_0}_{N,N;(m)}|\hat{H}|\Psi^{i_0;j_0}_{N,N;(m)}\rangle-E_N^{(m)}$ is given, for $m=3,5,7$, as $\tilde{\Delta}^{(m)}=\left[|F_1^C(m)|+\Delta F^C(m,1)+2|F_2^A(m)|-F_1^{(m)}\right]/m$, $\tilde{\Delta}^{(m)}=\Delta^{(m)}/(e^2/\varepsilon\ell_0)$, where

$$F_{\tilde{n}}^{(m)} = (1/\pi)\sum_{k=-\infty}^{\infty}e^{-\pi m\ (k-\tilde{n}/m)^2}\int_{-\infty}^{\infty}d\xi\int_0^\infty d\eta \times e^{-\eta^2/2}\ S_m^2(\xi)\ G_m(\xi,\eta;k-\tilde{n}/m), \quad (20)$$

$$\Delta F^C(m,\tilde{n})=-\frac{2^{3/2}}{\sqrt{\pi}m^{3/2}}\sum_{k=1}^{\infty}\frac{\exp(-\frac{\pi k^2}{m})}{k}\sin^2\left(\frac{\pi k\tilde{n}}{m}\right) +\frac{2}{\pi m}\int_0^\infty\int_0^\infty\frac{d\xi d\eta}{\sqrt{\xi^2+\eta^2}}\ S_m^2(\xi)\ G_m(\eta), \quad (21)$$

$G_m(\eta)=\exp(-\eta^2/4)\ [\exp(-\eta^2/4)-S_m(\eta)]$, and it is taken into account that only $\tilde{n}=1$ corresponds to $\Delta^{(m)}$. For $m=1$, $\Delta^{(1)}-|g_0|\mu_BB=(e^2/\varepsilon\ell_0)[|F_1^C(1)|+\Delta F^C(1,0)+2|F_2^A(1)|]$, where g_0 is the bare Landé g-factor. I calculate numerically that $\tilde{\Delta}^{(1)}-|g_0|\mu_BB/(e^2/\varepsilon\ell_0)\approx1.253895$ (i.e., very close to $\sqrt{\pi/2}\approx1.253314$), $\tilde{\Delta}^{(3)}\approx0.170657$, $\tilde{\Delta}^{(5)}\approx0.069867$, and $\tilde{\Delta}^{(7)}\approx0.036086$.

Point out that the ground-state Eq. (9) shows broken symmetry "liquid-crystal" behavior of 2DES as the electron density, Eq. (10), is periodic along y-direction, with period $\ell_0\sqrt{2\pi/m}$. We can make electron density much more homogeneous, however, the latter state has much higher energy than $U^C(m)$.

For the ground-state Eq. (9), at $\nu=1/m$, I calculate (details will be published elsewhere [22]) that the Hall conductance $\sigma_H=e^2/(2m\pi\hbar)$; i.e., it is properly quantized. Similar to Refs. [2, 24], we can speculate that for a weak disorder if the Fermi level still lies in a gap or mobility gap the Hall conductance should be quantized in a finite range of B.

Present energy gap $\tilde{\Delta}^{(3)}$ is about 1.6 times larger than typically calculated for the Laughlin liquid pertinent excitation gap [9, 11, 12]. For detailed comparison of the gap with experiment it is known that a finite thickness of 2DES should be taken into account as well as effects

of disorder [14, 25]. In addition, we can speculate that many-body effects similar to those studied in [26] (for "traditional" $\nu = 1$ state) and related with edge states here, maybe, also will lead to highly asymmetric pinning of the Fermi level within the energy gap. Then, similar to [26], actual activation gap can be much smaller than $\tilde{\Delta}^{(m)}/2$.

In summary, I have presented, at $\nu = 1/m$, the theory of liquid-crystal ground-state with periodic, along one direction, density of 2DES and uniform density of ions. The ground-state has strong correlations between 2DES and ions. The Hall conductance is properly quantized. Excitation gap, for $m = 1,\ 3,\ 5,\ 7$, is finite; quasielectron and quasihole charges are fractional, $\pm e/m$, for $m \geq 3$.

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